THEORETICAL INVESTIGATION OF THE MAIN SECTION OF A TURBULENT CIRCULAR JET IN STALLING FLOW

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A solution of the problem is proposed by an integral method. The results of a computation of trajectories, axial velocity, and jet boundaries by the method proposed for different expulsion angles agree with the test data.

Many papers that contain attempts to determine theoretically the characteristics of a circular jet in a transverse flow are known in the literature. This is associated with the extensive utilization of such jets in practice. A major part of these investigations is devoted to determining the jet trajectories (see [1]). Other jet characteristics are also determined in a number of papers (for instance, [2, 3]) by using modeling of circular jet development in a transverse flow by the development of vortex pairs since vortex motion is detected in tests in transverse jet sections, seemingly two vortices rotating in opposite directions. It must be noted that this approach permits giving qualitative more often than quantitative descriptions of the flow in the jet. Attempts at numerical integration of the motion and continuity equations for a three-dimensional flow [4, 5] show the possibility, in principle, of obtaining a computed flow pattern in the jet being developed in the transverse stream, that is qualitatively and to a certain degree quantitatively in agreement with test for a set of empirical constants usual for jet flows in a $k - \varepsilon$ model of turbulence [4] or under an assumption on the constancy of the turbulent viscosity coefficient [5]. Qualitative agreement between the computed and test flow patterns is obtained for a numerical integration of the Navier–Stokes equations in [6] in application to jet system outflow into a stalling stream bounded by channel walls under the assumption of flow laminarity.

The significant disagreement with test obtained in [4, 5] is visibly associated with the fact that the flow singularities at the nozzle exit and on the underlying surface around the jet that were detected in tests were not taken into account in giving the boundary conditions (see [7, 8], say). It must be emphasized that the influence of the stalling flow on the flow in the jet is detected in tests [8] not only at the nozzle exit but even within the nozzle.

Such an exact assignment of the characteristics is not required in solving the problem by an integral method. Satisfaction of the conditions "in the mean" is necessary since all the flow characteristics enter the equations under the integral sign. In this connection, there are attempts in the literature to determine circular jet characteristics in a stalling flow by integral methods (see [9, 10], say). In our opinion, analysis of these interesting researches is contained in [11]. It must be noted that the formulas in [9] permit determination just of the trajectory and axial velocity of the jet, require a significantly greater set of empirical constants here, and do not afford a passage to the limit to the ordinary submerged jet. The jet mass-flow characteristics do not agree with the test data.

The computed method [10] yields a better qualitative description of the flow pattern in the jet (including the isotach). However, the computed jet velocity and mass-flow characteristics profiles differ significantly from tests which is visibly associated with the fact that a number of assumptions and primarily the assumption about the ejection capacity of the jet in a stalling flow contracts the tests data.

A flow model in a circular jet in a transverse flow is proposed in [12] on the basis of experimental data known in the literature and the solution of the problem of the initial section of such a jet by an integral method is examined. It is here considered that the flow singularities in a circular jet in a stalling flow that are known from tests are re-

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lated to the stalling flow velocity component normal to the jet trajectory. It is assumed that this velocity component is proportional to the rate of change of the jet boundaries in the plane of symmetry and in the lateral direction resulting in deformation of the jet transverse section, and additional fluid mass flow ejectable from the surrounding space as compared with the submerged jet or a jet in a coflow. On the basis of this model, the solution of the problem of the main section of a circular jet in a transverse flow by an integral method is examined.

It must again be emphasized that utilization of an integral method in contrast to a numerical method permits "smoothing out" inaccuracies that originate upon replacement of the real substantially nonuniform velocity and pressure fields at the nozzle exit by uniform fields because these quantities enter under the integral sign. The solution of problems about the initial and main sections by using the physical flow model proposed in [12] permits a comparatively simple through-computation of the characteristics of a jet from the nozzle exit.

Let the jet emerge at an angle to the unlimited stream. Under the effect of the stalling flow the jet is curved and at a large distance from the nozzle exit its direction approaches the direction of the stalling flow. In conformity with the plane sections hypothesis it can be considered that the jet is developed in a flow of variable entanglement

$$\frac{u_\delta}{u_0} = \frac{V_\infty}{u_0}\cos\alpha \tag{1}$$

and around each of its sections goes a flow of velocity normal to the jet trajectory

$$V_{\infty n} = V_\infty \sin\alpha. \tag{2}$$

If the equation of the jet trajectory X = f(Y) is known, then the quantities $\sin\alpha$ and $\cos\alpha$ are easily determined by means of known formulas [13]. When solving the problem about the main jet section, just as in the initial section we will consider the lines of equal velocities, meaning the transverse jet section also to have the shape of an ellipse. Analysis of experimental data showed that in the jet section where one velocity maximum still exists, on the axis, the velocity profiles in jet radial sections are described completely satisfactorily by the Schlichting formula despite the fact that the jet edges in the main section are stalled all the more by the flow with distance from the nozzle exit and the section is transformed from elliptical into horseshoe-shaped. This affords a foundation for solving problems about the jet main section also by using an integral method. However, in contrast to the solution of the problem about the initial section it is already impossible to neglect the pressure change along the jet trajectory and it must be taken into account that the stalling flow velocity component directed along the jet trajectory is variable.

We will consider that the mean rarefaction over the section in the jet is proportional to the velocity head of the stalling flow velocity component normal to the jet trajectory since, as was noted above, it "corresponds" to the pressure on the jet contour, meaning the mean pressure in its section which should be below atmospheric since a reduced pressure exists on the major portion of the jet contour. Then

$$\frac{p}{\rho} = \frac{p_\infty}{\rho} - \varepsilon^2 \frac{V_{\infty n}^2}{2}. \tag{3}$$

Since we consider the velocity profiles known, then it is necessary to find the velocity change along the jet axis and halfwidth in the plane of symmetry and in the lateral direction for a complete description of the flow pattern. To determine these quantities we use the integral equation of momentum, the mass flow rate equation, and the equation of lateral broadening of the jet [12]:

$$\frac{d}{dx}\int_0^{\pi/2} d\varphi \int_0^{r_\delta(\varphi)} u(u-u_\delta)\,r\,dr = -\frac{1}{\rho}\int_0^{\pi/2} d\varphi \int_0^{r_\delta(\varphi)} \frac{\partial p}{\partial x}\,r\,dr - u_\delta' \int_0^{\pi/2} d\varphi \int_0^{r_\delta(\varphi)} u\,r\,dr, \tag{4}$$

$$Q = 4\int_0^{\pi/2} d\varphi \int_0^{r_\delta(\varphi)} u\,r\,dr = Q_0 + Q_{\text{ej}} + Q_{\text{add}} \tag{5}$$

$$\delta_z' = \delta_y' + 2\varepsilon \frac{V_{\infty n}}{u_m}. \tag{6}$$

It is considered in (5) that the fluid flow rate through the jet transverse section is comprised of the initial mass flow, the mass flow ejected by an ordinary jet in the co-flow, and an additional mass flow coming into the jet at the section between the nozzle exit and the given section because of the formation of an additional mixing zone.

In conformity with the flow model proposed in [12], this additional mass flow is determined by the formula

$$\frac{Q_{\text{add}}}{2\pi u_0 r_0^2} = \frac{Q_{\text{add i}}}{2\pi u_0 r_0^2} + \frac{\varepsilon}{\pi}\frac{V_\infty}{u_0}\int_{x_\text{i}}^{x}\delta_z \sin\alpha\, dx. \tag{7}$$

The mass flow ejected by the jet in the co-flow can be determined either by using $\delta^{(0)}$ and $u_m^{(0)}/u_0$ found from the known solution [1] or by using the solution of the problem by the integral method, in conformity with which the conservation equations of the excess jet momentum and the mass-flow equation are used to determine the axial velocity and radius of the jet. It can be considered here that the mass flow change in the main section of the jet is proportional to the jet excess axial velocity and radius while the proportionality factor is determined from test. In conformity with this representation, the magnitude of the mass flow ejected by the jet into the co-flow equals

$$\frac{Q_{\text{ej}}}{2\pi r_0^2 u_0} = q_0 + q_\text{i} + q_\text{t} + c_1\int_{Y_\text{t}}^{Y}\frac{\delta^{(0)}}{\sin\alpha}\left(\frac{u_m^0}{u_0} - \frac{u_\delta}{u_0}\right)dY, \tag{8}$$

$$q_0 + q_\text{i} = 0{,}5 + (0{,}09 + 0{,}292 m_\delta)\, c_\text{i}\frac{1-m_\delta}{1+m_\delta}x + (0{,}0314 + 0{,}0093 m_\delta - 0{,}073 m_\delta^2)\left(\frac{1-m_\delta}{1+m_\delta}\right)^2 c_\text{i}^2 x^2 +$$
$$+\frac{\varepsilon}{2\pi}\frac{V_{\infty n}}{u_0}\left\{x + \left[(0{,}27 - 0{,}079 m_\delta)\, c_\text{i}\frac{1-m_\delta}{1+m_\delta} + \frac{\varepsilon}{2}\frac{V_{\infty n}}{u_0}\right]x^2\right\}, \tag{9}$$

$$q_\text{t} = c_1\int_{x_\text{i}}^{x_\text{t}}\delta\left(1 - \frac{u_\delta}{u_0}\right)dx. \tag{10}$$

It is considered here that the axial velocity in the transition section equals the jet outflow velocity.

We give the velocity profile in (4) and (5) by using the Schlichting formula

$$\frac{u - u_\delta}{u_m - u_\delta} = (1 - \eta^{3/2})^2, \quad \eta = r/r_\delta(\varphi). \tag{11}$$

After some manipulations, the combined solution of (4)-(6) yields

$$\delta_y = \frac{\left(F_1^2 + 4\frac{\varphi(x)}{f_2}\right)^{1/2} - F_1}{2}, \quad \frac{u_m}{u_0} = 7{,}5\left(A + \sqrt{A^2 + 0{,}267B}\right), \tag{12}$$

$$A = 0{,}1286 T_1 + 0{,}0049\frac{u_\delta}{u_0}, \quad B = \left(0{,}371 T_1 + 0{,}0619\frac{u_\delta}{u_0}\right),$$

$$T_1 = \frac{\delta_{y\text{t}}\delta_{z\text{t}} f_{1\text{t}} + I_1}{\varphi(x)}, \quad I_1 = \frac{V_\infty}{u_0}\int_{x_\text{t}}^{x}\sin\alpha\,\delta_y\delta_z\left[\frac{V_\infty}{u_0}\frac{\varepsilon^2}{2}\cos\alpha + 0{,}1286\frac{u_m}{u_0} + 0{,}3714\frac{V_\infty}{u_0}\cos\alpha\right]\alpha'(x)\,dx, \tag{13}$$

$$\delta_z = \delta_y + F_1(x), \quad F_1(x) = \delta_{z\text{i}} - \delta_{y\text{i}} + 2\varepsilon\frac{V_\infty}{u_0}\int_{x_\text{i}}^{x}\frac{\sin\alpha}{u_m/u_0}dx, \tag{14}$$

$$\varphi(x) = q_0 + q_\text{i} + c_{1\text{t}}\left[\delta_{y\text{i}}(x_\text{t} - x_\text{i}) + y'_{1\text{i}}\frac{(x_\text{t} - x_\text{i})^2}{2}\right] - c_{1\text{t}}\frac{V_\infty}{u_0}\{\delta_{y\text{i}}(X_\text{t} - X_\text{i}) + y'_{1\text{i}}[X_\text{t}(x_\text{t} - x_\text{i}) - I_4]\} +$$
$$+\frac{\varepsilon}{2\pi}\frac{V_\infty}{u_0}\int_{Y_\text{i}}^{Y}\delta_z dY + c_1\int_{Y_\text{t}}^{Y}\frac{\delta^{(0)}}{\sin\alpha}\left(\frac{u_m^{(0)} - u_\delta}{u_0}\right)dY;$$

$$I_4 = \int_{Y_i}^{Y_t} X\sqrt{1+X'^2(Y)}\,dY, \tag{15}$$

$$\delta_y \delta_z f_2 = \varphi(x), \quad f_2 = 0{,}1286\,\frac{u_m}{u_0} + 0{,}3714\,\frac{u_\delta}{u_0}. \tag{16}$$

The system (12)-(16) was solved on an electronic computer by successive approximation. The quantity α was here determined from the jet trajectory equation being given in the form of a third degree polynomial $X = f(Y)$, whose coefficients were found from conditions at the nozzle exit (the trajector issues from a given point at a given angle to the flow and its radius of curvature is such that the transverse equilibrium condition $u_0{}^2/R_0 = (\partial p/\partial n)/\rho$ is satisfied in the plane of symmetry) and contained an additional constant which was determined from a comparison with test [14] for m = 0.1 at one point $X/r_0 = 50$.

Consequently, the jet trajectory takes the form

$$X = \operatorname{ctg}\alpha_0 Y + \frac{m^2(1+\Delta p)}{8\sin^4\alpha_0}\,Y^2 + 0{,}168m^3(1+\Delta p)^{3/2}Y^3. \tag{17}$$

As mentioned above, it must be noted that a large number of papers (see [1, 9, 15-18], say) is devoted to determining the trajectory of a circular jet in a transverse flow, in which the jet trajectory was determined, as a rule, by using a differential equation obtained from the equilibrium condition of all the forces acting on a jet element and certain assumptions. Taking account of the comparative awkwardness of these solutions and the need for a more simple description, if possible, of the jet trajectory in a broad range of geometric and mode parameters, it is deemed expedient to determine the trajectory in the form (17).

Compared in Fig. 1a are the computed jet trajectories with tests for $\alpha_0 = 90°$ and for different magnitudes of the ratio between the stalling flow and jet velocities. It is seen from the figure that agreement between the computed curves and the experimental data is completely satisfactory in the whole investigated range of variation of the quantity m.

Computed jet trajectories determined by means of (17) for different blowing angles and m = 0.1 are compared with test data [14] in Fig. 1b. It is seen that the proposed formula describes the jet trajectory satisfactorily even for different blowing angles.

The computed curves u_m/u_0 are compared in Fig. 2a with test data [7, 14-16, 20-22] and new data of the authors obtained during an investigation of the main section of a circular jet in a stalling flow (the experimental set-up and method of conducting the experiment are analogous to those described in [7]).

It is seen from Fig. 2b that the agreement between computations by the method cited above and experiment is better than the computations executed in [4].

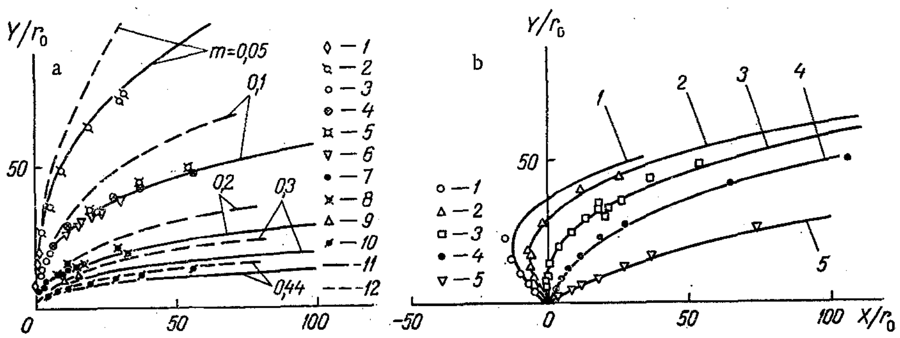

Fig. 1. Comparison of results of computing the jet trajectory with experiment: a) $\alpha_0 = 90°$: 1, 2) m = 0.05; 3-6) 0.1; 7, 8) 0.2; 9) 0.3; 10) 0.44; 11, 12) computation (2, 5, 8 – data of [14]; 6 – [15]; 12 – [1]; 1, 3, 4, 7, 9, 10, 11 – data of the authors); b) m = 0.1: 1) $\alpha_0 = 135°$; 2) 120; 3) 90; 4) 60; 5) 30°; points are data of [14], curves are computation.

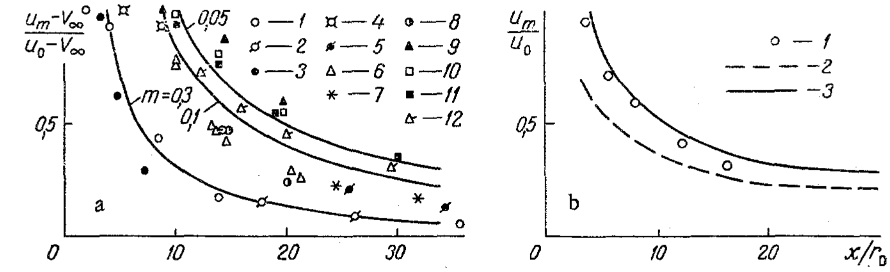

Fig. 2. Comparison of the results of computing the jet axial velocity with experiment (α_0 = 90°): a) 1-3) m = 0.3; 4-8) 0.1; 9-12) 0.05 (1, 4, 9 – data of [7]; 6, 10, 11 – [14]; 7 – [15]; 3 – [16]; 2, 5 – [20]; 8 – [21]; 12 – [22]); b) m = 0.25: 1) data of [19]; 2) [4]; 3) data of the authors.

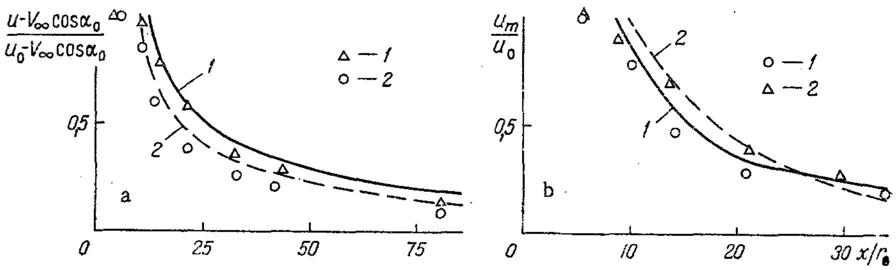

Fig. 3. Comparison of the results of computing the jet axial velocity with experiment for different blowing angles α_0: a) α_0 < 90°: 1) α_0 = 30°; 2) 60 (data of [14]); b) α_0 > 90°: 1) α_0 = 120°; 2) 150 (data of [14]).

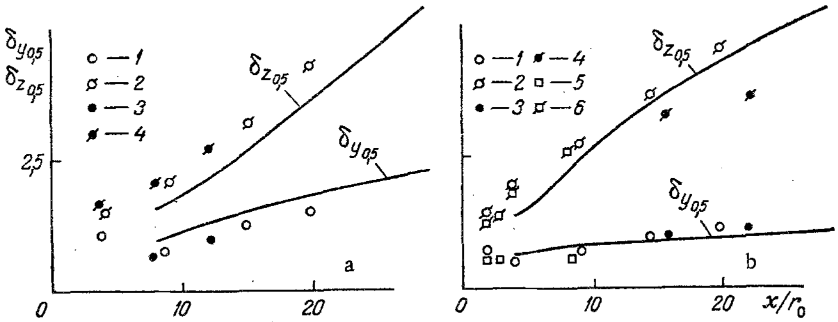

Fig. 4. Comparison of the results of computing the jet boundary with experiment: a: 1, 2) m = 0.1 (data of [7]); 3, 4) 0.135 (data of [20]); curves are computation (m = 0.1); b: 1, 2) m = 0.3 (data of [7]); 3, 4) 0.28 (authors' data); 5, 6) 0.333 (data of [20]); curves are computation (m = 0.3).

Shown in Fig. 3 is a comparison of the computed axial velocity with test data of [14] for different blowing angles and m = 0.1. Let us note that in conformity with [11] it was assumed that $u_\delta = 0$ in computations of the jet characteristics for blowing angles greater than 90° on sections where the jet is developed in a counter flow ($u_\delta < 0$).

A comparison is performed in Fig. 4 for the jet boundaries δ_y and δ_z. It is seen that agreement between the computed and experimental data is completely satisfactory.

Therefore, the flow model proposed in [12] and the method of computation elucidated above permit description of the flow characteristics of a circular jet in a stalling flow even in the main section.

NOTATION

c, c_1, c_i, c_{1t}, empirical constants; $c_1 = 0.1286c$, $c_{1t} = 0.069\, c_i$; m, ratio between the stalling flow and jet velocities; $m_\delta = u_\delta/u_0$; n, normal to the jet trajectory; p, static pressure; p_a, p_0, static pressures directly before and after the jet; p_∞, static pressure in the flow far ahead of the jet; Q, mass flow rate; Q_0, initial mass flow rate; $q_i = Q_{eji}/2\pi u_0 r_0^2$ and $q_t = Q_{ejt}/2\pi u_0 r_0^2$, relative fluid flow rates ejected in the initial and transition sections; r_0, jet radius at the nozzle exit; R_0 radius of curvature of jet trajectory at nozzle exit; $r_\delta(\varphi)$, radius of the jet boundary; r, φ, polar coordinates; u, longitudinal velocity component (in the direction of the jet trajectory); u_m, u_δ, velocities on the jet axis and on its boundary; u_0, mean mass flow rate of the jet outflow; V_∞, stalling flow velocity; $V_{\infty n}$, stalling flow velocity component normal to the jet trajectory; x, y, Cartesian coordinates associated with the jet trajectory; x, directed along the jet axis; y, perpendicular to x; y_{1p}', slope of the jet boundary at the end of the initial section; X, Y, Cartesian coordinates associated with the nozzle exit; X, directed along the stalling flow; Y, perpendicular to X; α, angle between the tangent to the jet trajectory and the stalling flow direction; α_0, jet blowing angle; $\Delta p = 2(p_\infty - p_0)/\rho V_\infty^2 = 1$; δ_y, δ_z, jet halfwidths in the plane of symmetry and in the transverse direction; $\varepsilon = 0.8$, an empirical constant. Subscripts: i is for values of the quantities at the end of the initial section; and t at the end of the transition section.

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